

# Yoshizawa's cross-helicity effect and its quenching

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A central quantity in mean-field magnetohydrodynamics is the mean electromotive force  $\overline{\mathcal{E}}$ , which in general depends on the mean magnetic field. It may however have a part independent of the mean magnetic field. Here we study an example of a rotating conducting body of turbulent fluid with non-zero cross-helicity, in which a contribution to  $\overline{\mathcal{E}}$  proportional to the angular velocity occurs (Yoshizawa 1990). If the forcing is helical, it also leads to an  $\alpha$  effect, and large-scale magnetic fields can be generated. For not too rapid rotation, the field configuration is such that Yoshizawa's contribution to  $\overline{\mathcal{E}}$  is considerably reduced compared to the case without  $\alpha$  effect. In that case, large-scale flows are also found to be generated.

*Keywords:* Mean-field dynamo; Rotating turbulence; cross-helicity effect; alpha effect

## 1 Introduction

Many studies of the large-scale magnetic fields in turbulent astrophysical bodies such as the Sun or the Galaxy are carried out in the framework of mean-field electrodynamics (see the textbooks by Moffatt 1978, Parker 1978, Krause and Rädler 1980, Zeldovich *et al.* 1983). It is based on the induction equation governing the magnetic field  $\mathbf{B}$ ,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} - \eta \mu_0 \mathbf{J}) , \quad (1)$$

where  $\mathbf{U}$  is the fluid velocity,  $\mathbf{J} = \nabla \times \mathbf{B} / \mu_0$  the current density,  $\eta$  the magnetic diffusivity, and  $\mu_0$  the vacuum permeability. Both the magnetic field  $\mathbf{B}$  and the velocity field  $\mathbf{U}$  are considered sums of mean parts,  $\overline{\mathbf{B}}$  and  $\overline{\mathbf{U}}$ , defined as proper averages of the original fields, and fluctuations. The averages are assumed to satisfy the Reynolds averaging rules. The mean magnetic field  $\overline{\mathbf{B}}$  then obeys the mean-field induction equation

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \overline{\mathcal{E}} - \eta \mu_0 \overline{\mathbf{J}}) . \quad (2)$$

Here  $\overline{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}}$  is the mean electromotive force resulting from the fluctuations of velocity and magnetic field,  $\mathbf{u} = \mathbf{U} - \overline{\mathbf{U}}$  and  $\mathbf{b} = \mathbf{B} - \overline{\mathbf{B}}$ . Generally,  $\overline{\mathcal{E}}$  can be represented as a sum

$$\overline{\mathcal{E}} = \overline{\mathcal{E}}^{(0)} + \overline{\mathcal{E}}^{(B)} \quad (3)$$

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of a part  $\bar{\mathcal{E}}^{(0)}$ , which is independent of  $\bar{\mathbf{B}}$ , and a part  $\bar{\mathcal{E}}^{(B)}$  vanishing with  $\bar{\mathbf{B}}$ . In many representations and applications of mean-field electrodynamics the part  $\bar{\mathcal{E}}^{(0)}$  of  $\bar{\mathcal{E}}$  is ignored. Only the part  $\bar{\mathcal{E}}^{(B)}$ , which is of crucial importance for dynamo action, is taken into account.

Here we focus our attention on the part  $\bar{\mathcal{E}}^{(0)}$  of  $\bar{\mathcal{E}}$ . It may depend on non-magnetic quantities influencing the turbulence, in general also on  $\bar{\mathbf{U}}$ . If the magnitude of  $\bar{\mathbf{U}}$  is small, and if  $\bar{\mathbf{U}}$  varies only weakly in space and time, we may write

$$\bar{\mathcal{E}}_i^{(0)} = \bar{\mathcal{E}}_i^{(00)} + \Xi_{ij}\bar{U}_j + \Upsilon_{ijk}\bar{U}_{j,k} \quad (4)$$

with  $\bar{\mathcal{E}}_i^{(00)}$  as well as  $\Xi_{ij}$  and  $\Upsilon_{ijk}$  being independent of  $\bar{\mathbf{U}}$ . Of course, the contribution  $\bar{\mathcal{E}}^{(00)}$  to  $\bar{\mathcal{E}}^{(0)}$  can only be non-zero if the turbulence allows us to define a direction. For example, turbulence in a rotating body shows in general an anisotropy determined by the angular velocity  $\boldsymbol{\Omega}$ , and  $\bar{\mathcal{E}}^{(00)}$  might then be proportional to  $\boldsymbol{\Omega}$ , say equal to  $c_\Omega\boldsymbol{\Omega}$ . The  $\Xi_{ij}$  term in (4) can only be unequal to zero if the turbulence lacks Galilean invariance. In the case of isotropic turbulence it describes a contribution to  $\bar{\mathcal{E}}^{(0)}$  proportional to  $\bar{\mathbf{U}}$ , say equal to  $c_U\bar{\mathbf{U}}$ . Note that in forced turbulence Galilean invariance can be broken if, independent of the flow, the forcing is fixed in space and shows a finite correlation time (for an example see Rädler and Brandenburg 2010). The  $\Upsilon_{ijk}$  term, if restricted to isotropic turbulence, corresponds to a contribution to  $\bar{\mathcal{E}}^{(0)}$  proportional to  $\nabla \times \bar{\mathbf{U}}$ , say equal to  $c_W\nabla \times \bar{\mathbf{U}}$ . The coefficients  $c_\Omega$  and  $c_W$  are, in contrast to  $c_U$ , pseudo scalars. The contributions  $c_\Omega\boldsymbol{\Omega}$  and  $c_W\nabla \times \bar{\mathbf{U}}$  to the mean electromotive force were first considered by Yoshizawa (1990). He found that both  $c_\Omega$  and  $c_W$  are closely connected with the cross helicity  $\overline{\mathbf{u} \cdot \mathbf{b}}$ . In what follows the occurrence of the contributions  $c_\Omega\boldsymbol{\Omega}$  and  $c_W\nabla \times \bar{\mathbf{U}}$  to the mean electromotive force  $\bar{\mathcal{E}}$  is called “Yoshizawa effect”. This effect has been invoked to explain magnetic fields in accretion discs (Yoshizawa and Yokoi 1993) and spiral galaxies (Yokoi 1996). It has also been used to explain the surprisingly high level of magnetic fields in young galaxies (Brandenburg and Urpin 1998), because the amplification of the mean field by this effect is independent of any seed magnetic field owing to the field’s linear as opposed to exponential growth with time.

As for the part  $\bar{\mathcal{E}}^{(B)}$  of  $\bar{\mathcal{E}}$ , we recall here the traditional ansatz

$$\bar{\mathcal{E}}^{(B)} = \alpha_{ij}\bar{B}_j + \eta_{ijk}\bar{B}_{j,k}. \quad (5)$$

It can be justified for cases in which  $\bar{\mathbf{B}}$  varies only weakly in space and time. In the simple case of isotropic turbulence it takes the form  $\bar{\mathcal{E}}^{(B)} = \alpha\bar{\mathbf{B}} - \eta_t\nabla \times \bar{\mathbf{B}}$ , which describes the  $\alpha$  effect and the occurrence of a turbulent magnetic diffusivity (Krause and Rädler 1980).

In this paper we report on numerical simulations of magnetohydrodynamic turbulence in a rotating body, that is, under the influence of the Coriolis force. We present results for the mean electromotive force and discuss them in the light of the above remarks, focussing particular attention on the Yoshizawa effect.

## 2 Model

We consider forced magnetohydrodynamic turbulence of an electrically conducting, compressible, rotating fluid which is permeated by a magnetic field. An isothermal equation of state is used so that the pressure  $p$  and the mass density  $\rho$  are proportional to each other,  $p = \rho c_s^2$ , with  $c_s$  being a constant sound speed. The magnetic field  $\mathbf{B}$ , the fluid velocity  $\mathbf{U}$  and the mass density  $\rho$  are assumed to obey

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} - \eta\mu_0\mathbf{J} + \mathbf{f}_M, \quad (6)$$

$$\frac{D\mathbf{U}}{Dt} = -c_s^2\nabla \ln \rho - 2\boldsymbol{\Omega} \times \mathbf{U} + \frac{1}{\rho}\mathbf{J} \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot 2\rho\nu\mathbf{S} + \mathbf{f}_K, \quad (7)$$

$$\frac{D \ln \rho}{Dt} = -\nabla \cdot \mathbf{U}. \quad (8)$$

Unless indicated otherwise, we exclude a homogeneous part of the magnetic field.  $\mathbf{A}$  is the magnetic vector potential,  $\nabla \times \mathbf{A} = \mathbf{B}$ , and  $\eta$  again the magnetic diffusivity,  $D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$  is the advective time derivative,  $\boldsymbol{\Omega}$  the angular velocity which defines the Coriolis force,  $S_{ij} = \frac{1}{2}(U_{i,j} + U_{j,i}) - \frac{1}{3}\delta_{ij}\nabla \cdot \mathbf{U}$  the trace-less rate of strain tensor,  $\nu$  the kinematic viscosity, while  $\mathbf{f}_M$  and  $\mathbf{f}_K$  define the magnetic and kinetic forcing specified below. The simultaneous magnetic and kinetic forcing is a simple way to generate non-zero cross helicity. We admit only small Mach numbers, that is, only weak compressibility effects.

The equations (6)–(8) are numerically solved in a cubic domain with the edge length  $L$  assuming periodic boundary conditions. Then  $k_1 = 2\pi/L$  is the smallest possible wavenumber. We assume that  $\boldsymbol{\Omega}$  is parallel to the positive  $z$  direction, that is,  $\boldsymbol{\Omega} = (0, 0, \Omega)$  with  $\Omega > 0$ .

The forcing is assumed to be constant during each time step but changes from one step to the next (Brandenburg 2001). We define it until further notice by

$$\mathbf{f}_M = N_M \text{Re}\{\tilde{\mathbf{f}}_{\mathbf{k}(t)} \exp[i\mathbf{k}(t) \cdot \mathbf{x} + i\phi(t)]\}, \quad \mathbf{f}_K = N_K \text{Re}\{i\mathbf{k}(t) \times \tilde{\mathbf{f}}_{\mathbf{k}(t)} \exp[i\mathbf{k}(t) \cdot \mathbf{x} + i\phi(t)]\}, \quad (9)$$

with

$$N_M = \mathcal{N}_M c_s \sqrt{\mu_0 \rho_0 c_s / k_f \delta t}, \quad N_K = \mathcal{N}_K c_s \sqrt{c_s / k_f \delta t}, \quad (10)$$

where  $\mathcal{N}_M$  and  $\mathcal{N}_K$  are dimensionless amplitudes,  $\rho_0$  is the initial mass density, considered as uniform,  $k_f$  the average forcing wavenumber and  $\delta t$  the duration of the time step. Furthermore,

$$\tilde{\mathbf{f}}_{\mathbf{k}(t)} = \frac{\mathbf{f}_{\mathbf{k}(t)} - i\varepsilon \hat{\mathbf{k}}(t) \times \mathbf{f}_{\mathbf{k}(t)}}{\sqrt{1 + \varepsilon^2}}, \quad (11)$$

where  $\hat{\mathbf{k}}(t) = \mathbf{k}(t)/|\mathbf{k}(t)|$  is the unit vector of  $\mathbf{k}(t)$  and  $\mathbf{f}_{\mathbf{k}(t)}$  is a statistically homogeneous isotropic non-helical random vector field (Haugen *et al.* 2004). Both forcing functions  $\mathbf{f}_M$  and  $\mathbf{f}_K$  are non-helical if  $\varepsilon = 0$ , and maximally helical if  $|\varepsilon| = 1$ . They are  $\delta$ -correlated in time, so for a discrete time step  $\delta t$  they must be proportional to  $\delta t^{-1/2}$ . Owing to  $\delta$ -correlation, the fluid flow in our model is Galilean invariant. We put

$$\mathbf{f}_{\mathbf{k}(t)} = \frac{\mathbf{k}(t) \times \mathbf{e}(t)}{\sqrt{(\mathbf{k}(t)^2 - (\mathbf{k}(t) \cdot \mathbf{e}(t))^2) \delta t}}, \quad (12)$$

where  $\mathbf{e}(t)$  is a random unit vector not parallel to  $\mathbf{k}(t)$ . In this way we have  $\nabla \cdot \mathbf{f}_M = \nabla \cdot \mathbf{f}_K = 0$ . The wavevectors  $\mathbf{k}$  are chosen such that their moduli  $k = |\mathbf{k}|$  lie in a band of width  $\delta k$  around a mean forcing wavenumber  $k_f$ , that is,  $k_f - \delta k \leq k \leq k_f + \delta k$ , and we choose  $\delta k = k_1$ .

We describe our simulations using the magnetic Prandtl number  $\text{Pr}_M$ , the Coriolis number  $\text{Co}$ , the magnetic Reynolds number  $\text{Re}_M$ , and the Lundquist number  $\text{Lu}$ ,

$$\text{Pr}_M = \nu/\eta, \quad \text{Co} = 2\Omega/u_{\text{rms}}k_f, \quad \text{Re}_M = u_{\text{rms}}/\eta k_f, \quad \text{Lu} = b_{\text{rms}}/\sqrt{\mu_0 \rho_0 \eta} k_f, \quad (13)$$

with  $u_{\text{rms}}$  and  $b_{\text{rms}}$  being defined using averages over the full computational volume. While  $\text{Pr}_M$  and  $\text{Co}$  are input parameters,  $\text{Re}_M$  and  $\text{Lu}$  are used for giving results. For our numerical simulations we use the PENCIL CODE<sup>1</sup>, which is a high-order public domain code (sixth order in space and third order in time) for solving partial differential equations, including the hydromagnetic equations given above.

<sup>1</sup><http://pencil-code.googlecode.com/>

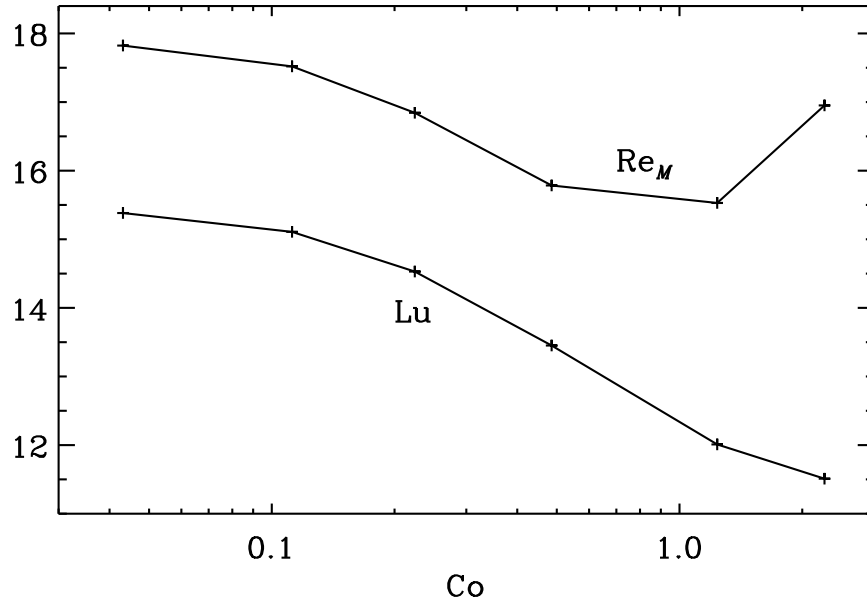


Figure 1. Non-helical case. Dependence of  $\text{Re}_M$  and  $\text{Lu}$  on  $\text{Co}$ .

### 3 Results and Interpretation

We have performed a series of simulations with  $\text{Pr}_M = 1$ ,  $\mathcal{N}_K = 0.01$ ,  $\mathcal{N}_M = 0.005$ ,  $k_f = 5k_1$  and varying  $\text{Co}$ . As initial conditions we used  $\mathbf{U} = \mathbf{A} = \mathbf{0}$  and  $\rho = \rho_0$ .

We discuss the results here in terms of space averages taken over the full computational volume defined above. These averages, which we denote by angle brackets, are of course independent of space coordinates. We put first  $\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{b}$  and  $\mathbf{U} = \langle \mathbf{U} \rangle + \mathbf{u}$ . Using  $\mathbf{B} = \nabla \times \mathbf{A}$  and the periodicity of  $\mathbf{A}$ , we have  $\langle \mathbf{B} \rangle = \mathbf{0}$ , that is,  $\mathbf{B} = \mathbf{b}$ . By contrast,  $\langle \mathbf{U} \rangle$  is not necessarily equal to zero.  $\langle \mathbf{B} \rangle = \mathbf{0}$  is however enough to justify  $\langle \mathbf{U} \cdot \mathbf{B} \rangle = \langle \mathbf{u} \cdot \mathbf{b} \rangle$  and  $\langle \mathbf{U} \times \mathbf{B} \rangle = \langle \mathbf{u} \times \mathbf{b} \rangle$ .

Within this framework the mean electromotive force discussed above and denoted there by  $\overline{\mathcal{E}}$  is equal to  $\langle \mathbf{u} \times \mathbf{b} \rangle$ . According to the ideas expressed in the Introduction, and recalling that volume averages of spatial derivatives of our periodic variables  $\mathbf{A}$  or  $\mathbf{U}$  vanish, we expect

$$\langle \mathbf{u} \times \mathbf{b} \rangle = c_\Omega \mathbf{\Omega} + c_U \langle \mathbf{U} \rangle \quad (14)$$

with  $c_\Omega$  determined by the cross-helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$ . Owing to Galilean invariance of the flow in our model  $c_U$  should vanish. In all simulations under the mentioned conditions  $\langle \mathbf{U} \rangle$  turned out very small. Even if the initial condition for  $\mathbf{U}$  was changed and so larger  $|\langle \mathbf{U} \rangle|$  were generated, no influence of  $\langle \mathbf{U} \rangle$  on  $\langle \mathbf{u} \times \mathbf{b} \rangle$  was observed. We conclude from this that indeed  $c_U = 0$ .

Let us give further results first for non-helical forcing,  $\varepsilon = 0$ . In this case we expect no  $\alpha$  effect and see no reason for the generation of large-scale magnetic fields. Figure 1 gives  $\text{Re}_M$  and  $\text{Lu}$ , considered as measures for  $u_{\text{rms}}$  and  $b_{\text{rms}}$ , as functions of  $\text{Co}$ . Figure 2 shows that the cross helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$  and, if  $\text{Co} \neq 0$ , also the  $z$  component of the mean electromotive force  $\langle \mathbf{u} \times \mathbf{b} \rangle$  are non-zero. The moduli of the  $x$  and  $y$  components of  $\langle \mathbf{u} \times \mathbf{b} \rangle$  are negligible. According to Yoshizawa's result we expect  $\langle \mathbf{u} \times \mathbf{b} \rangle_z = \frac{1}{2} \zeta \langle \mathbf{u} \cdot \mathbf{b} \rangle \text{Co}$  with  $\zeta$  being a number of the order of unity. Figure 3 shows that  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle \text{Co}$  is indeed around 0.5.

Consider next the case of maximally helical forcing,  $\varepsilon = 1$ . The simulations for this case have been carried out with a modified definition of  $\mathbf{f}_K$ . In (9) and (10),  $i\mathbf{k}(t) \times \tilde{\mathbf{f}}_{\mathbf{k}(t)}$  has been replaced by  $\tilde{\mathbf{f}}_{\mathbf{k}(t)}$ , and  $\sqrt{c_s/k_f \delta t}$  by  $\sqrt{c_s k_f / \delta t}$ . Now an  $\alpha$  effect is to be expected and, as a consequence, the generation of magnetic fields with scales comparable to that of the computational domain (Brandenburg 2001). Indeed, as illustrated by Figure 4, different types of large-scale magnetic fields with a dominant wavenumber  $k = k_1$  occur. Following Hubbard *et al.* (2009), we call them “meso-scale fields”. As can be seen in the example of

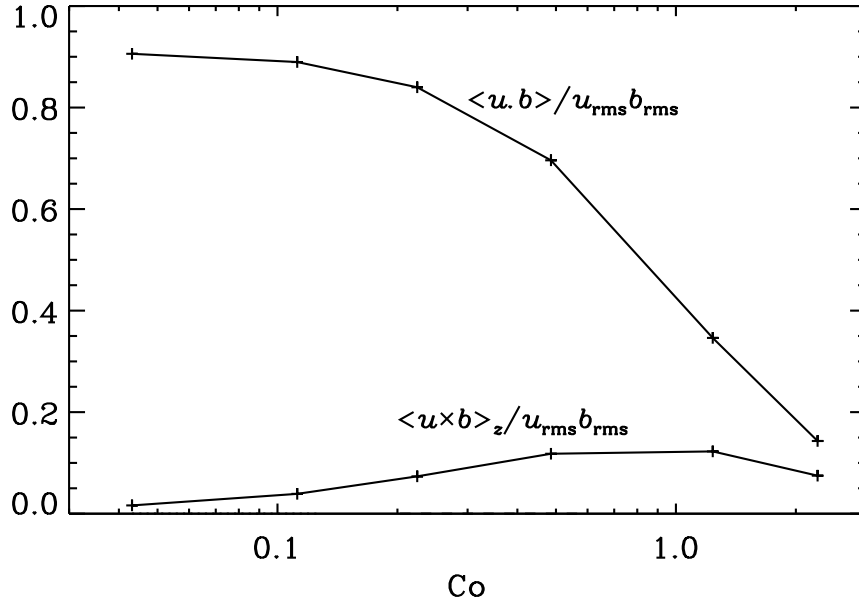


Figure 2. Non-helical case. Normalized cross helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle / u_{\text{rms}} b_{\text{rms}}$  and  $z$  component of normalized mean electromotive force  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / u_{\text{rms}} b_{\text{rms}}$  as functions of  $\text{Co}$ ; the moduli of the  $x$  and  $y$  components of  $\langle \mathbf{u} \times \mathbf{b} \rangle / u_{\text{rms}} b_{\text{rms}}$  are below  $10^{-3}$ .

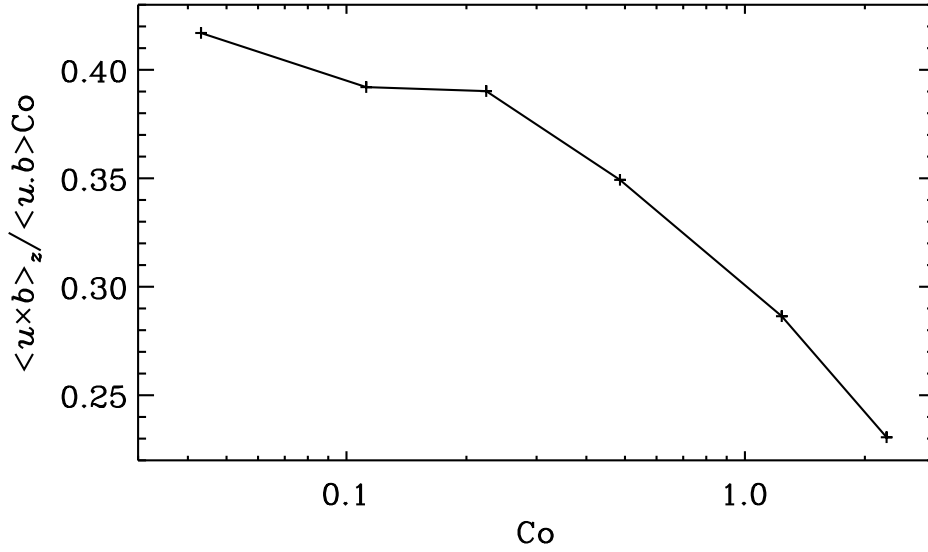


Figure 3. Non-helical case. Dependence of  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle \text{Co}$  on  $\text{Co}$ .

Figure 5, these fields are to a good approximation of Beltrami shape. Three different types of such fields have been observed,

$$\mathbf{B}^X = B_0(0, \sin k_1 x, \cos k_1 x), \quad \mathbf{B}^Y = B_0(\cos k_1 y, 0, \sin k_1 y), \quad \mathbf{B}^Z = B_0(\sin k_1 z, \cos k_1 z, 0), \quad (15)$$

in general with common phase shifts of the components in the  $x$ ,  $y$  and  $z$  directions.  $B_0$  was always of the order of several equipartition values  $B_{\text{eq}}$ , defined by  $B_{\text{eq}} = \sqrt{\mu_0 \rho_0} u_{\text{rms}}$ . For not too large  $\text{Co}$  all three types,  $\mathbf{B}^X$ ,  $\mathbf{B}^Y$  and  $\mathbf{B}^Z$ , turned out to be possible, but for  $\text{Co}$  exceeding a value of about unity only that of type  $\mathbf{B}^Z$  occurs. This becomes understandable when considering that for the amplification of meso-scale fields of type  $\mathbf{B}^X$  and  $\mathbf{B}^Y$ , the products  $\alpha_{yy}\alpha_{zz}$  and  $\alpha_{xx}\alpha_{zz}$  are important, while for  $\mathbf{B}^Z$  it is  $\alpha_{xx}\alpha_{yy}$ , but  $|\alpha_{zz}|$  is reduced by rotational quenching (Rüdiger 1978) for large values of  $\text{Co}$ .

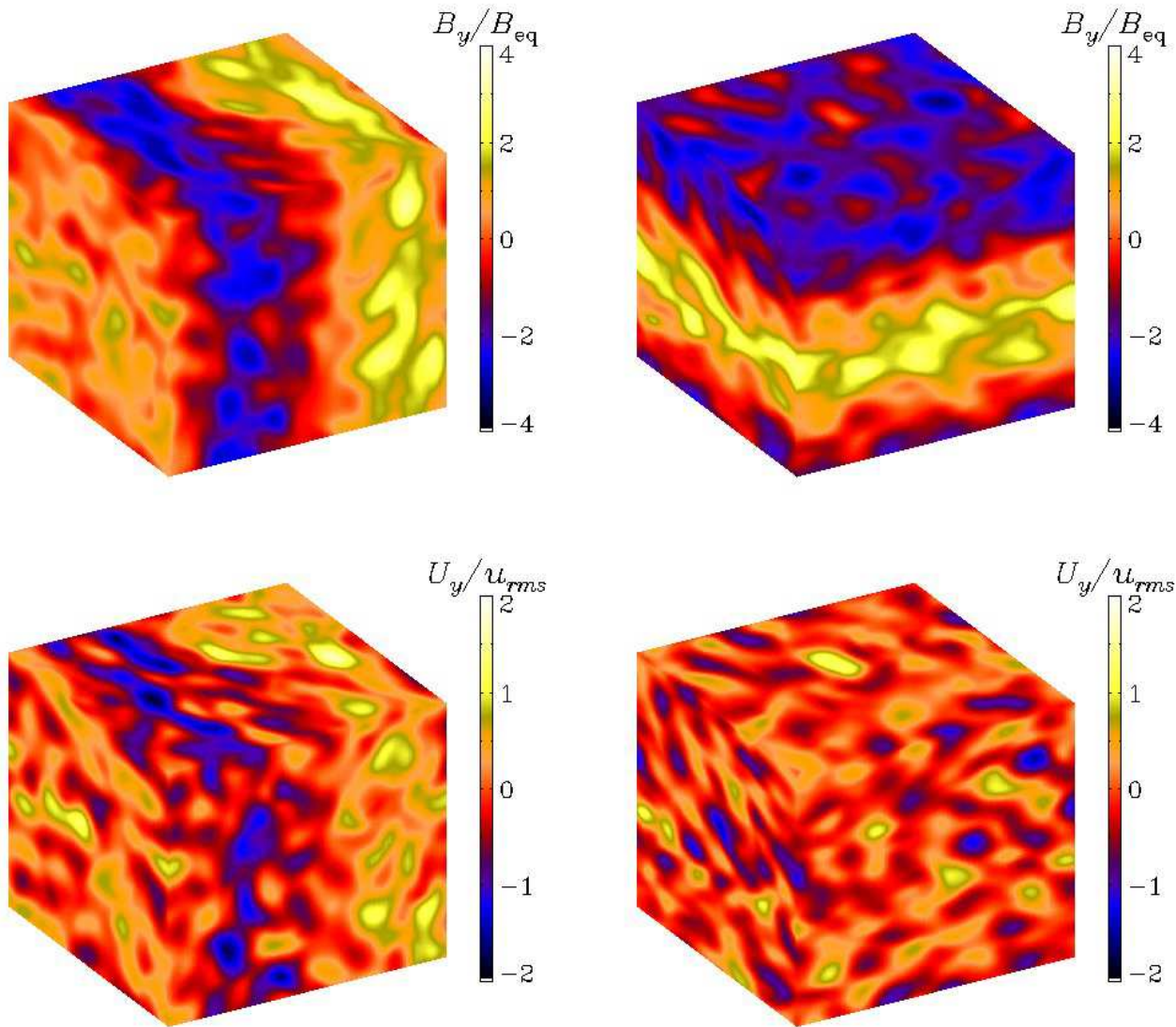


Figure 4. Helical case. Upper row:  $B_y/B_{eq}$  on the periphery of the computational domain, state with  $B^X$  type field (left) and  $B^Z$  type field (right),  $Co = 0.37$ . Lower row: same as above, but  $U_y/u_{rms}$ .

Furthermore, meso-scale flows of type  $U^X$  and  $U^Y$ , defined analogously to (15), are also possible; see the lower panels of Figure 4. Such flows have never been seen in the absence of cross helicity. However, already for small  $Co$  it seems impossible to tolerate  $U^Z$  flows. This might be connected with the fact that the Coriolis force acting on a  $U^Z$  flow would produce a  $90^\circ$  phase-shifted flow proportional to  $(\cos k_1 z, -\sin k_1 z, 0)$ . By comparison, the Coriolis force acting on a  $U^X$  or a  $U^Y$  flow gives another one proportional to  $(\sin k_1 x, 0, 0)$  or  $(0, \cos k_1 y, 0)$ , respectively, which does not directly interfere with  $U^X$  or  $U^Y$ .

Both the cross helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$  and the mean electromotive force  $\langle \mathbf{u} \times \mathbf{b} \rangle$  are influenced by the presence of the meso-scale magnetic fields and meso-scale flows. Figure 6 shows the dependence of  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$  and  $\langle \mathbf{u} \times \mathbf{b} \rangle_z$  on the types of the meso-scale magnetic fields and on  $Co$ . Meso-scale magnetic fields of  $B^X$  or  $B^Y$  type together with meso-scale flows enhance the level of  $\langle \mathbf{u} \cdot \mathbf{b} \rangle / u_{rms} b_{rms}$ , especially for small values of  $Co$ . With meso-scale magnetic fields of  $B^Z$  type  $\langle \mathbf{u} \cdot \mathbf{b} \rangle / u_{rms} b_{rms}$  is reduced relative to that in the non-helical case (Figure 2), because  $b_{rms}$  is enhanced by a factor of about 2. As Figure 7 demonstrates,  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle Co$  depends now crucially on whether meso-scale fields of  $B^X$  or  $B^Y$  type or of  $B^Z$  type are present. In the first case the Yoshizawa effect is clearly reduced by the meso-scale fields; in the second case it is enhanced

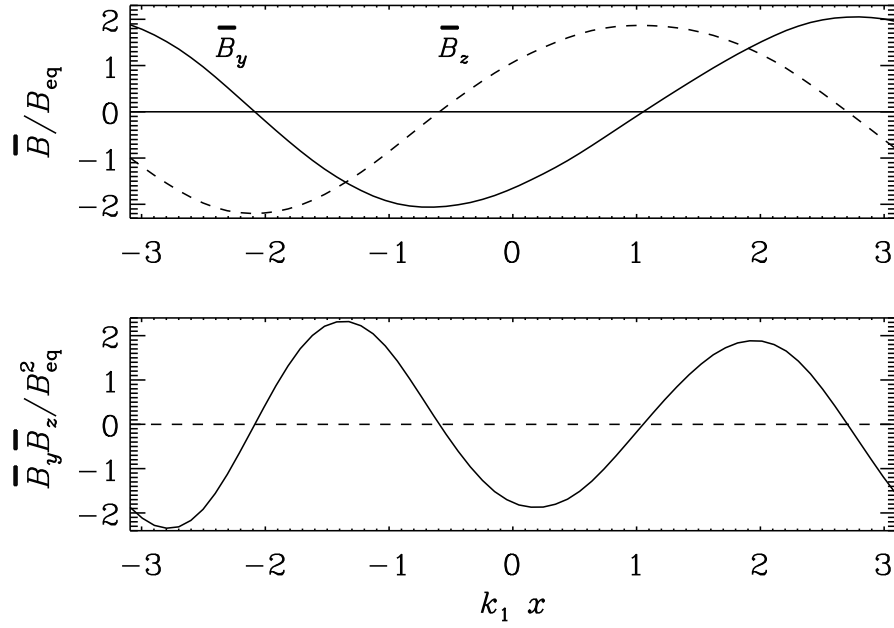


Figure 5. Helical case. Profiles of  $\bar{B}_y(x)/B_{eq}$  and  $\bar{B}_z(x)/B_{eq}$  as well as their product in a state with  $\mathbf{B}^Z$  field,  $Co = 0.2$ . Overbars denote  $yz$  averages. The dashed line gives the level of the  $x$  average of  $\bar{B}_y \bar{B}_z / B_{eq}^2$ , which is close to zero (here,  $\approx -10^{-3}$ ).

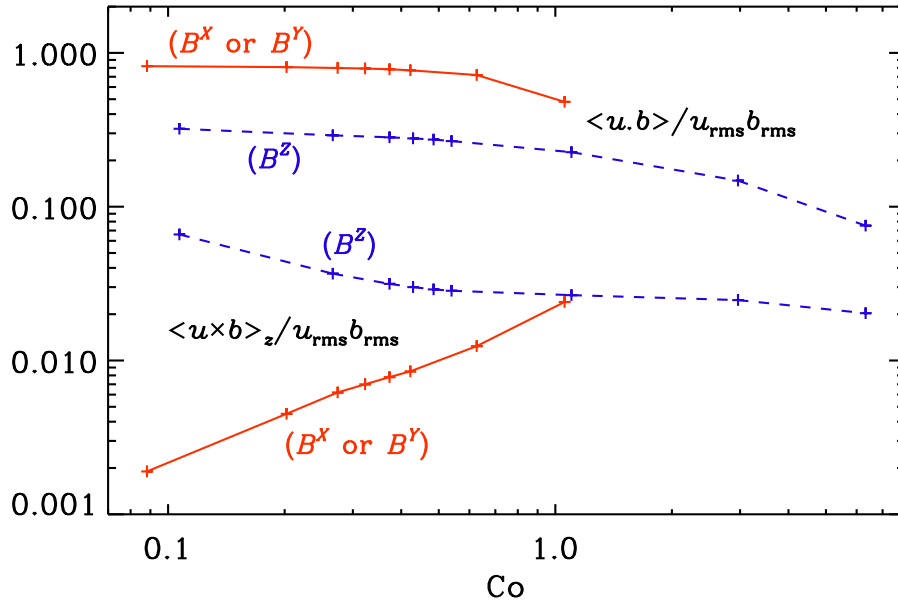


Figure 6. Helical case. Normalized cross helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle / u_{rms} b_{rms}$  (upper lines) and  $z$  component of the normalized mean electromotive force  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / u_{rms} b_{rms}$  (lower lines) as functions of  $Co$ ; the moduli of the  $x$  and  $y$  components are below  $10^{-3}$ . Solid lines correspond to states with  $\mathbf{B}^X$  or  $\mathbf{B}^Y$  type fields, dashed lines to states with  $\mathbf{B}^Z$  type fields.

for small  $Co$ , but reduced for larger  $Co$ .

The remarkable strength of the meso-scale fields can lead to strong magnetic quenching effects. As a first approach to the understanding of such effects the non-helical case has been studied with an imposed homogeneous magnetic field in the  $y$  or  $z$  directions,  $(0, B_0, 0)$  or  $(0, 0, B_0)$ , respectively. Figure 8 shows as an example the dependence of  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle Co$  at  $Co \approx 0.25$  on  $B_0/B_{eq}$ . It suggests that in the helical case the reduction of  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle Co$  by  $\mathbf{B}^X$  or  $\mathbf{B}^Y$  fields, which possess a non-zero  $z$  component, is stronger than that by  $\mathbf{B}^Z$  fields, which have no  $z$  components.

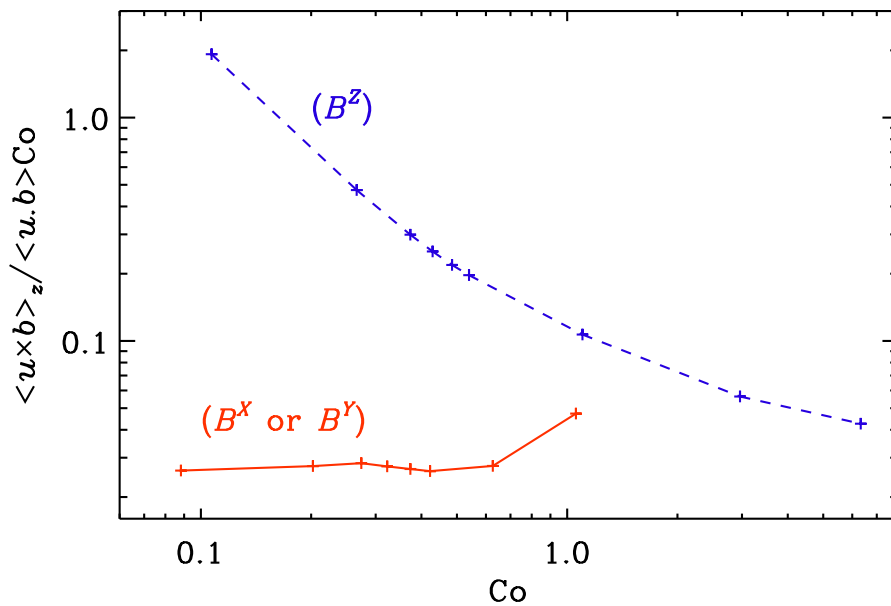


Figure 7. Helical case. Dependence of  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle Co$  on  $Co$ . Solid lines correspond to states with  $B^X$  or  $B^Y$  type fields, dashed lines to states with  $B^Z$  type fields.

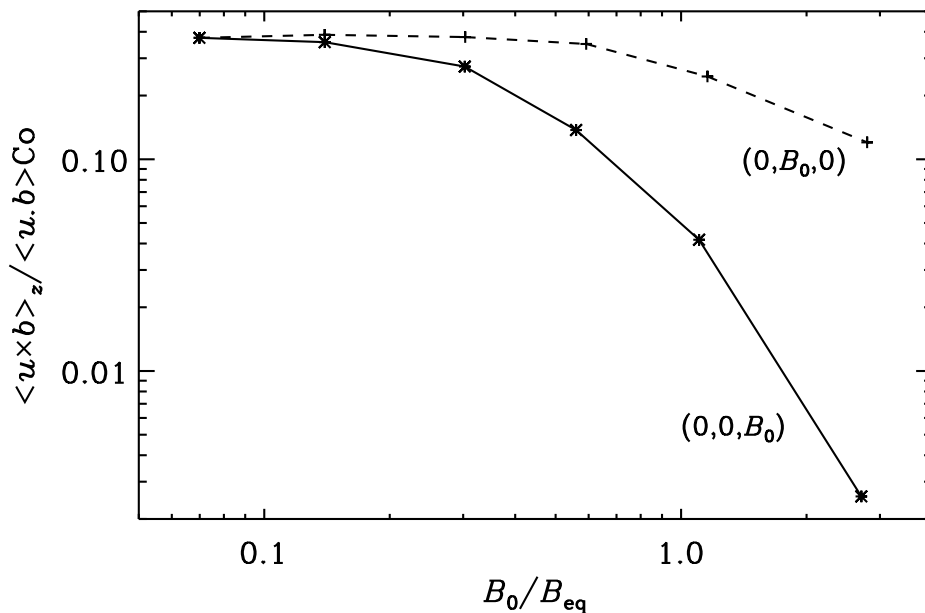


Figure 8. Non-helical case with an imposed homogeneous magnetic field in  $y$  or in  $z$  direction,  $(0, B_0, 0)$  (dashed line) or  $(0, 0, B_0)$  (solid line). Dependence of  $\langle \mathbf{u} \times \mathbf{b} \rangle_z / \langle \mathbf{u} \cdot \mathbf{b} \rangle Co$  on  $B_0/B_{eq}$  at  $Co \approx 0.25$ ,  $Re_M \approx 10$ .

#### 4 Discussion

The mean electromotive force in a turbulent fluid may have a part which is independent of the mean magnetic field and also independent of the mean flow. As an example for that we have studied forced hydromagnetic turbulence in a rotating body. In this case the Yoshizawa effect occurs, that is, a contribution  $c_\Omega \Omega$  to  $\langle \mathbf{u} \times \mathbf{b} \rangle$ . We have confirmed that  $c_\Omega$  is determined by the mean cross-helicity  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$ . We have also seen that, if an  $\alpha$  effect is present, the Yoshizawa effect can to a large extent be compensated by the action of magnetic fields maintained by this  $\alpha$  effect.

In astrophysics, the occurrence of non-zero cross-helicity is not a very common phenomenon. We give



here a few examples in which the findings of this paper could be of interest. In the solar wind the systematic radial flow together with the Sun's large-scale magnetic field give rise to cross helicity of opposite sign in the two hemispheres. Although this primarily implies cross helicity associated with mean flow and mean magnetic field, it also results in cross helicity associated with the fluctuations. Together with the Sun's rotation, the latter should then produce a component of the mean electromotive force that is distinct from that related to the  $\alpha$  effect. Note, however, that the cross-helicity associated with the fluctuations is directly a consequence of the cross helicity from the large-scale field.

Another example where small-scale cross helicity can be generated is in a stratified layer with a vertical magnetic field (Rüdiger *et al.* 2011). Again, the sign of  $\langle \mathbf{u} \cdot \mathbf{b} \rangle$  is linked to the orientation of the large-scale field relative to the direction of gravity.

Finally, cross helicity can be generated spontaneously and can then be of either sign, such as in the Archontis dynamo (Archontis 2000); for kinematic simulations see Archontis *et al.* (2003), Cameron and Galloway (2006). Sur and Brandenburg (2009) have analyzed this dynamo with respect to the Yoshizawa effect. In this example too, large-scale and small-scale fields are intimately related. This interrelation means that whenever we expect the  $\bar{\mathcal{E}}^{(0)}$  term to be present in an astrophysical system, there should also be a mean magnetic field. Such an effect that is odd in the mean magnetic field might therefore instead just as well be associated with an  $\alpha$  effect. As it turns out, this is also the case in the present simulations, where a large-scale magnetic field has been produced. In the present case, we have gone a step further by including also kinetic helicity, in addition to just cross helicity. This produces an  $\alpha$  effect and, as a consequence of this, a large-scale magnetic field. This field is particularly important when rotation is weak, because then the Yoshizawa effect is strongly quenched by this field.

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